LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2013**

MT 5406 - COMBINATORICS

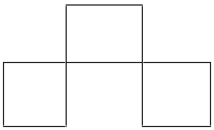
Date : 18/11/2013 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

SECTION-A

ANSWER **ALL** THE QUESTIONS: **(10x2 = 20)**

1. State exclusion principle.
2. Define Stirling number of second kind. S_3^2-
3. How many 7 letter words of binary digits are there?
4. In an examination a candidate has to pass in each of the five papers. How many different combinations of papers are there so that a student may fail?
5. Define recurrence relation.
6. Find the co-efficient of x^5 in $(1 + x)(1 + 2x)(1 + 3x)(1 + 7x)(1 + 4x)$.
7. Find the Rook polynomial for the chess board C in the diagram below,



8. Define derangement.
9. Find Φ(100).
10. Define cycle index of a permutation group.

SECTION-B

ANSWER ANY **FIVE** QUESTIONS: **(5x8 = 40)**

11. There are 30 females , 35 males in a junior class while there are 25 females and 20 males in the senior class. In how many ways can a committee of 10 be chosen so that there are exactly 5 females and 3 juniors in the committee?
12. Prove that the number of distributions of n distinct objects into m distinct boxes with the objects in each box arranged in a definite order is the rising factorial $[m]^n$.
13. Derive the Pascal's identity using the concept of generating functions.
14. (i) Define exponential generating function.
    (ii) Find the number of *r* letter sequences that can be formed using the letters *P, Q, R* and *S* such that in each sequence, there are an odd number of *P's* and an even number of *Q's*. (2 + 6)
15. State and prove Multinomial theorem .
16. Define permanent and find the permanent of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. (2–6)
17. State and prove Burnside Frobenius theorem.
18. Describe the problem of Fibonacci with an illustration.

SECTION-C

ANSWER ANY **TWO** QUESTIONS (2x20 = 40)

19. (i) Derive the recurrence formula for S_n^m . Formulate a table for S_5^5.
(ii) If there exists a bijection between the set of n-letter words with distinct letters out of an alphabet of m letters and the set of n-tuples on m letters, without repetitions, then show that the cardinality of each of these sets is m(m-1)(m-2)…(m-n+1). (10 + 10)

20. (i) Prove that the element f of R[t] given by $f(t) = \sum_{k=0}^{\infty} a_k t^k$ has an inverse in R[t] if and only if α_0 has an inverse in R.
(ii) Determine the OGF of the sequence $\{(r+n-1)C_{(n-1)}\}, r \geq 0$ by differentiation of infinite geometric series. (10+10)

21. Find the elements of the group G of symmetries of a square. Enumerate the different types of colorings of the 4 vertices, if each vertex is to be either red or blue. Exhibit in a diagram the patterns. Also find Pattern Inventory of G.

22. State and solve Ménage problem.